Planckian Energy Scattering of D-branes and M(atrix) Theory in Curved Space

I.V.Volovich
Steklov Mathematical Institute
Gubkin St.8, GSP-1, 117966, Moscow, Russia

Abstract

We argue that black p-branes will occur in the collision of D0-branes at Planckian energies. This extents the Amati, Ciafaloni and Veneziano and 't Hooft conjecture that black holes occur in the collision of two light particles at Planckian energies. We discuss a possible scenario for such a process by using colliding plane gravitational waves. D-branes in the presence of black holes are discussed. M(atrix) theory and matrix string in curved space are considered. A violation of quantum coherence in M(atrix) theory is noticed.

1 Introduction

It was suggested [1] that the large N limit of the dimensional reduction of the ten dimensional $U(N)$ supersymmetric Yang-Mills theory to one dimension might be interpreted as eleven dimensional M-theory in light cone gauge. The reduction of the supersymmetric Yang-Mills theory to two dimensions was interpreted as matrix string theory [2]. This is based on the observation [3] that the effective low energy theory of N coincident parallel Dirichlet branes is described by the dimensional reduction of the $U(N)$ supersummetric Yang-Mills theory.

M(atrix) theory [1] is described by the following Lagrangian

$$L = \frac{1}{2} tr[\dot{Y}^i \dot{Y}^i + \frac{1}{2}[Y^i, Y^j]^2 + 2\theta^T \dot{\theta} + 2\theta^T \gamma_i [\theta, Y^i]] \tag{1.1}$$

where Y^i are Hermitian $N \times N$ matrices while θ is a 16-component fermionic spinor each component of which is an Hermitian $N \times N$ matrix and $i, j = 1, ..., 9$.

In [4, 5] the interaction between D-branes in the non-relativistic approximation has been considered. It was found [5] that supergravity is valid at distances greater than the string scale, while the description in terms of gauge theory in flat space is valid in the sub-stringy domain.

The application of the flat background is justified if the curvature is small. However if one has D-branes in the presence of a black hole we have to take into account the

non-trivial background. The low energy bulk theory of D-branes is the supersymmetric Yang-Mills theory coupling with supergravity in ten dimensions. In this note we discuss the interpretation of the dimensional reduction of this theory as M-theory (if $p=0$) or matrix string (if $p=1$) in curved background. We will argue that (black) p-branes will occur in the collision of D0-branes at Planckian energies.

2 M(atrix) theory in curved space

The low energy effective theory of D-branes in Minkowski spacetime is given by the dimensional reduction of the supersymmetric gauge theory in the ten dimensional Minkowski spacetime [3]. If one has D-branes in Minkowski spacetime but in curved coordinates we have to start from the supersymmetric Yang-Mills theory in curved coordinates. Then we get a version of the M(atrix) theory Lagrangian (1.1) in the curved coordinates. If one has D-branes in a curved spacetime, for instance D-branes in the presence of black hole then it is natural to expect that the low energy effective theory will be given by the dimensional reduction of the supersymmetric gauge theory coupling with supergravity in the ten dimensional curved spacetime.

Let us consider the Yang-Mills theory in the D-dimensional space-time with the metric g_{MN}. The action is

$$I=-\frac{1}{4}tr\int d^Dx\sqrt{g}F_{MN}F_{PQ}g^{MP}g^{NQ} \tag{2.2}$$

where $F_{MN}=i[D_M,D_N],\quad D_M=\nabla_M-iA_M$. Let $\gamma:X^M=X^M(\sigma),\ \sigma=(\sigma_0,...,\sigma_p)$ be a $p+1$-dimensional submanifold (p-brane) and let us consider the dimensional reduction to γ. One has $A_M=(A_\alpha,\ Y_i),\alpha=0,...,p;\ i=p+1,...,D-1$ and $F_{MN}=(F_{\alpha\beta},F_{\alpha i},F_{ij}),\ g^{MN}=(g^{\alpha\beta},g^{\alpha i},g^{ij})$. The Lagrangian is

$$L=-\frac{1}{2}tr[D_\alpha Y_iD_\beta Y_jg^{\alpha\beta}g^{ij}-\frac{1}{2}[Y_i,Y_j][Y_m,Y_n]g^{im}g^{jn}+...] \tag{2.3}$$

Here $Y_i=Y_i(X^P(\sigma)),\ g_{MN}=g_{MN}(X^P(\sigma))$. If one takes $p=1$ then the Lagrangian (2.3) describes the matrix string [2, 6, 7] in curved background. For a 0-brane $X^M=X^M(\tau)$ in the gauge $A_0=0,\ g^{0i}=0$ one has

$$L=-\frac{1}{2}tr[\dot{Y}_i\dot{Y}_jg^{00}g^{ij}-\frac{1}{2}[Y_i,Y_j][Y_m,Y_n]g^{im}g^{jn}] \tag{2.4}$$

The Lagrangian (2.4) describes the bosonic part of M(atrix) theory in curved space. It can be reduced to the bosonic part of (1.1) if we take $g^{00}=-1,\ g^{ij}=\delta^{ij}$ and $X^M(\tau)=\tau\delta_{M0}$.

Notice that in contrast to the picture with a noncommutative geometry in the short distance regime in [1] here in fact we have the classical commutative spacetime coordinates X^M. The matrices Y_i describe the noncommutative dynamical system in the ordinary classical spacetime. The corresponding Hamiltonian is

$$H=\frac{1}{2}tr[P^iP^jg_{ij}-\frac{1}{2}[Y_i,Y_j][Y_m,Y_n]g^{im}g^{jn}] \tag{2.5}$$

One deals with quantum mechanics in the dependent on time background $g^{ij}(\tau)=g^{ij}(X(\tau))$. Now the properties of the matrix quantum mechanics depend on the choice of

the curve $X(\tau)$. The one-loop effective action for the theory (2.4) with the p-brane metric g_{MN} can be computed using the background field method by the standard procedure. One gets corrections to the phase shift δ obtained in [5, 19, 11] which are now under consideration. If one takes a geodesic near the singularity then generically one gets the creation of particles (D0-branes) [8].

If the metric g_{ij} in (2.4) describes a black hole then one can apply to M(atrix) theory the known Hawking arguments [9] on the violation of quantum coherence. If one has M(atrix) theory in flat spacetime (1.1) then of course there exists the unitary evolution operator and there is no a violation of quantum coherence. But in this case simply there is no problem for discussion because there are no black holes in the flat spacetime.

To get the supersymmetric M(atrix) theory or matrix string theory in curved background one has to take the dimensional reduction of the supersymmetric Yang-Mills theory in ten dimensions coupling with supergravity,

$$L = \frac{1}{4g^2\Phi}tr(F_{MN}^2) - \frac{1}{2}tr(\overline{\chi}\Gamma^M D_M\chi) - \frac{1}{2\kappa^2}R - \frac{3\kappa^2}{8g^4\Phi^2}H_{MNP}^2 + ... \tag{2.6}$$

For the p-brane

$$ds^2 = f^{-\frac{1}{2}}(-dt^2 + dx_1^2 + ... + dx_p^2) + f^{\frac{1}{2}}(dx_{p+1}^2 + ... + dx_9^2), \tag{2.7}$$

$f = 1 + \frac{q}{r^{7-p}}$, which is the BPS-state we don't expect the Hawking radiation but there is the spacetime singularity here and there is back reaction of the gas of D-branes to the metric which is described by the equations:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} =< T_{\mu\nu} > \tag{2.8}$$

where $T_{\mu\nu}$ is the energy-momentum tensor of D-branes.

It seems that M(atrix) theory being quantum mechanics in the curved spacetime suffers from the well known problems such as non-controllable spacetime singularities and the violation of quantum coherence. There is a hope [10, 11] that large N limit gauge theory might be used to study these problems.

3 Scattering of D-branes and creation of black holes

Amati, Ciafaloni and Veneziano [12] and 't Hooft [13] have argued that at extremely high energies interactions due to gravitational waves will dominate all other interactions. They conjectured that black holes will occur in the collision of two light particles at Planckian energies with small impact parameter. In [12, 14] the elastic scattering amplitude in the eikonal approximation was found in the form

$$A(s,t) \propto s\int d^2b e^{iqb}e^{iI_{cl}} \tag{3.9}$$

Here s and t are the Mandelstam variables and b is the impact parameter. I_{cl} was taken to be the value of the boundary term for the gravitational action calculated on the sum of two Aichelburg-Sexl shock waves,

$$I_{cl} = Gs\log b^2 \tag{3.10}$$

The action I_{cl} is equal to the phase shift $\delta(b, v)$ for the process of the elastic scattering, see [14, 15].

One cannot see the creation of black holes in this approximation. In [15] the following mechanism of the creation of black holes in the process of collisions of the Planckian energy particles has been suggested. Each of the two ultrarelativistic particles generates a plane gravitational wave. Then these plane waves collide and produce a singularity or black hole. The phase of the transition amplitude from plane waves to black holes was calculated as the value of the action on the corresponding classical solution.

It seems that scattering of D-branes at extremely high energies and small impact parameter will be similar to the described picture. In particular two colliding D0-branes at small impact parameter should produce p-branes.

Scattering of D-branes at large impact parameter has been considered in [16, 17, 18, 5, 19]. The 0-brane metric lifted to 11 dimensions is

$$ds^2 = dudv + (1 + \frac{q}{r^7})du^2 + dx_1^2 + ... + dx_9^2 \tag{3.11}$$

It represents a plane-fronted wave moving in the x_{11} direction. At long distances the gravitational wave can be considered as a plane wave. Plane wave solutions in supergravity have been considered in [20, 21, 22, 23, 24]. Collision of two plane gravitational waves produce a spacetime that is locally isometric to an interior of black hole, see [15]. To estimate the amplitude for the creation of black holes one can use the expression

$$A \propto \int d^9 b e^{iqb} e^{iI_{cl}} \tag{3.12}$$

where I_{cl} is the value of the boundary term for the gravitational action of the $D = 11$ supergravity [25] calculated on the solution describing colliding plane waves [15].

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References

[1] T.Banks, W.Fischler, S.Shenker and L.Susskind, hep-th/9608086

[2] L.Motl, hep-th/9701025

[3] E.Witten, hep-th/9510135

[4] U.H.Danielsson, G.Ferretti and B.Sundborg, hep-th/9603081

[5] M.Douglas, D.Kabat, P.Pouliot and S.Shenker, hep-th/9608024

[6] T.Banks and N.Seiberg, hep-th/9702187

[7] R.Dijkgraaf, E.Verlinde and H.Verlinde, hep-th/9703030

[8] N.D.Birrell and P.C.W.Davies, *Quantum fields in curved space*, Cambridge University Press, 1982

[9] S.Hawking, Phys.Rev.D14 (1976) 2460

[10] I.V.Volovich, hep-th/9608137

[11] M.Douglas, J.Polchinski and A.Strominger, hep-th/9703031

[12] D.Amati, M.Ciafaloni and G.Veneziano, Phys. Let. B197 (1987) 81; Nucl. Phys. B347(1990)

[13] G. 't Hooft, Nucl.Phys. B 304 (1988) 867

[14] M.Fabbrichesi, R.Pettorino, G.Veneziano and G.A.Vilkovisky, Nucl. Phys. B419 (1994) 147

[15] I. Ya. Aref'eva, K.S.Viswanathan and I.V.Volovich, Nucl. Phys. B 452 (1995)346; B462(1996)613; hep-th/9412157

[16] C.Bachas, hep-th/9511043

[17] S.Gubser, I.Klebanov, A.Hashimoto and J.Maldacena, hep-th/9601057

[18] G.Lifschytz, hep-th/9604156

[19] V.Balasubramanian and F.Larsen, hep-th/9703039

[20] G.W.Gibbons, Nucl.Phys. B207 (1982) 337

[21] R.Guven, Phys. Let. 191 (1987)265

[22] E.Bergshoeff, R.Kallosh and T.Ortin, hep-th/9406009

[23] G.T.Horowitz and A.A.Tseytlin, hep-th/9407099

[24] J.G.Russo and A.A.Tseytlin, hep-th/9611047

[25] P.Horava and E.Witten, hep-th/9603142